THE LAWRENCE SCHOOL, LOVEDALE
SUBJECT ENRICHMENT ACTIVITY - JUNE 2019
CLASS 12 (MATHEMATICS)

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1. If $tan^{-1}x + tan^{-1}y = \frac{\pi}{4}$, then write the value of $x + y + xy$
2. Solve for x: $2\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$
3. Simplify: $\boldsymbol{cot^{-1}\frac{1}{\sqrt{x^2-1}} \quad for\ x < -1}$
4. Differentiate the following with respect to x:
   $$sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$$
5. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1,$ prove that
   $$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$
6. Find the inverse of matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ and hence show that $A^{-1}.A = I$
7. Find $\frac{dy}{dx}$, $if \quad x^y + y^x + x^x = a^b$
8. If $x\sin(a+y) + \sin a\cos(a+y) = 0$, Prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$
9. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1600. School B wants to spend ₹ 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.
10. Find the inverse of the following matrix using elementary operations:
    $$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$
11. Using properties of determinants, prove that
    $$\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(a+c)^2}{b} \end{vmatrix} = 2(a+b+c)^3$$
12. If f(x) is defined by the following function

$$f(x) = \begin{cases} \dfrac{\sin(a-1)x + sinx}{x} & if\ x < 0 \\ c & if\ x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & if\ \ x > 0 \end{cases}$$

is continuous at x= 0, find the values of a,b and c

13. If x = Sint , y = Sin(kt) , prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$

14. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$,then a) find $|5AB|$ b) $|adjA|$

15. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

Find AB and use this result in solving the following system of equations

x – y + z = 4 , x – 2y – 2z = 9 , 2x + y + 3z = 1

16. If $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{0} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{-1} & \mathbf{0} \end{pmatrix}$, $\boldsymbol{find\ A^2 - 5A + 4I}$ and hence find a matrix X such that $\boldsymbol{A^2 - 5A + 4I + X = 0}$.

17. Using properties of determinants, prove that :

$$\begin{vmatrix} \mathbf{2y} & \boldsymbol{y - z - x} & \mathbf{2y} \\ \mathbf{2z} & \mathbf{2z} & \boldsymbol{z - x - y} \\ \boldsymbol{x - y - z} & \mathbf{2x} & \mathbf{2x} \end{vmatrix} = \boldsymbol{(x + y + z)^3}$$

18. Differentiate $tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ $with\ respect\ to\ cos^{-1}\left(2x\sqrt{1-x^2}\right)$ when $x \neq 0$

19. Differentiate $sin^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{2}\right)$ w.r.t x

20. The function f(x) is defined as follows:

$$f(x) = \begin{cases} x^2 + ax + b\ ; & 0 \le x < 2 \\ 3x + 2\ ; & 2 \le x \le 4 \\ 2ax + 5b\ ; & 4 < x \le 8 \end{cases}$$

If f(x) is continuous in [0, 8], find the values of ‘a’ and ‘b’.

21. Using properties of determinants, prove that

$$\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c)$$

22. Discuss the continuity and differentiability of the function $f(x) = |x| + |x-1|$ in the interval $(-1, 2)$.

23. If $y = \sqrt{sinx + \sqrt{sinx + \sqrt{sinx + \cdots \ to \ \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

24. Differentiate: $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}(x^2)$.

25. If $\sin[cot^{-1}(x+1)] = \cos(tan^{-1}x)$, then find x.

26. If $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}, \ then \ find \ x.$

27. If $x^y = e^{x-y}, prove \ that$,Prove that $\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$

28. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$

29. Using properties of determinants prove that

$$\begin{vmatrix} \mathbf{1+a} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1+b} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1+c} \end{vmatrix} = \mathbf{abc}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{a}} + \frac{\mathbf{1}}{\mathbf{b}} + \frac{\mathbf{1}}{\mathbf{c}}\right)$$

30. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

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